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# GRINDING KINETICS OF SELECTED MINERALS WITH REFERENCE TO THE NUMBER OF CONTACT POINTS 

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#### Abstract

Results of experiments whose aim was to determine the rate of grinding of mineral materials with different susceptibility to comminution are discussed in the paper. Experimental materials were gabbro and sandstone. The experiments were carried out in a laboratory ball mill. During grinding the number (mass) of balls of constant diameter at constant feed mass was changed. The main objective of the experiments was to give a specific rate of grinding of particular size fractions of the mineral materials used in road making. An initial equation was the Gardner and Austin formula in a differential form for discrete values of the fractions, assuming an ideal mixing of the ground material. The grinding rate for selected size fractions of the ground material was described by a correlation equation. Parameters of the correlation equation and the effect of ball number (mass) on their values were specified. Additionally, a mathematical relation describing the change of a mean particle diameter in time, was given.


Keywords: ball mill, contact point, grinding rate

## INTRODUCTION

Due to a complex character of the motion of feed and grinding elements in the mill, and variable in time contribution of particular grinding mechanisms related to it, the description of process kinetics requires refined mathematical tools. In the Department of Process Equipment researches are carried out on the modelling of grinding kinetics, which will enable to scale them up from laboratory to pilot-plant or industrial scale. One of possible solutions to simplify the description of grinding kinetics in ball mills is the use of a possible relation between the rate of grinding of particular material fractions and the number of contact points between the balls.

The process of grinding proceeds mainly due to a complex interaction between the balls and ground material and additionally by the action of grinding elements on the

[^0]inner drum surface. Feed, which is between the balls, is subjected to attrition and shearing with a possible addition of crushing [MATTAN 1971, LYNCH 1974]. These mechanisms of comminution appear mainly in a cataracting motion of the balls. At a cascading motion of the balls, there is additionally an impact mechanism involved which is a result of collisions of balls falling on the bed at the bottom of the grinding chamber. This type of motion, at which impacts prevail, occurs for rotations frequency of the mill close to a critical frequency. This is a very desirable phenomenon because of grinding intensity, but dimensions of industrial ball mills and related inertia forces reduce the character of mill operation at velocities close to the critical frequency [SHIPWAY 1993]. Due to this, choosing lower frequencies of the mill rotations, the contribution of particular grinding mechanisms can be changed by changing the size and number of balls. During the grinding process, the mill filling with feed and the number of balls in the drum can be changed. Owing to limited mill dimensions, it is desirable to determine such composition of balls at which grinding of particular size fractions takes place at the highest possible rates. A ground product characterised by very high monodispersivity is obtained. It is also important to obtain a specified mean particle diameter after a possibly shortest grinding time.

An increase of ball sizes determines an increment of a single ball mass and an increase of mutual interaction between the grinding elements. An increasing size of grinding media, at unchanged mill filling with balls, provokes a decrease of the number of contact points. This causes a decrease of mini-regions, where stresses destroying ground material particles can occur in a given moment. Selection of ball diameter depends on the strength of ground material and on particle diameter. In the case of bigger particles that require stronger breaking forces, balls of bigger size should be used, while for smaller particles and weaker materials better effects will be obtained when the number of points of ball contacts will be increased, hence, by increasing their number at the cost of diameter [HEIM et al. 2004].

A simple construction of the mill does not keep up with the efficiency of grinding process. Low process efficiency makes technologists look for such a composition of balls and packing of the mill, at which the mean particle diameter decrease is the fastest. This will enable a more economic use of the mill operating time [HEIM OLEJNIK and PAWLAK 2005].

From this point of view grinding effects in a mill with different number of balls were analysed. Dry grinding process was carried out in a laboratory mill. A feed was a mixture of mineral materials used in road pavement production.

## EXPERIMENTAL

Changes in particle size distribution of ground material in time were investigated in a laboratory mill of inner diameter 0.25 m and total water volume of $6 \mathrm{dm}^{3}$. Grinding was a dry process. Experimental material included gabbro and sandstone. Prior to grinding, the minerals were subjected to preliminary processing that included selection
of particles with diameters ranging from 1 to 2.5 mm . Grinding was performed using four types of drum filling with corrundum balls 30 mm in diameter. Process parameters are given in Table 1.

Grinding was carried out until the moment when on the sieve with 1 mm mesh size, less than 2.5 g material remained (this was about $1.4 \%$ mass of input fraction - 180 g ). In determined time intervals samples were taken to analyse particle size distribution. The analysis was made using an ANALYSETTE 22 laser particle size analyser (FRITSCH).

Table 1. Equipment and process parameters of grinding

| Trial <br> no. | Number/mass of balls <br> $[-/ \mathrm{kg}]$ | Frequency of mill <br> revolutions $\left[\mathrm{min}^{-1}\right]$ | Feed type | Feed volume $\left[\mathrm{dm}^{3}\right]$ |
| :---: | :---: | :---: | :---: | :---: |
| A | $14 / 0.901$ |  |  |  |
| B | $19 / 1.15$ |  |  | 0.6 |
|  | C | $23 / 1.392$ |  |  |

RESULTS AND DISCUSSION
Table 2 gives examples of particle size analysis of $1 \mathrm{dm}^{3}$ ground gabbro at mill filling with the balls of mass 0.901 kg .

Grinding rates for particular size fractions were calculated basing on the particle size analysis. Equation (1) proposed by Gardner and Austin in a differential form for discrete values of fractions, was used in calculations assuming an ideal mixing of the ground material.

$$
\begin{equation*}
\frac{d w_{i}(t)}{d t}=-S_{i} w_{i}(t)+\sum_{j=1, i>1}^{i-1} S_{j} b_{i, j} \cdot w_{j}(t) \tag{1}
\end{equation*}
$$

To determine the distribution function $\mathrm{b}_{\mathrm{i}, \mathrm{j}}$ the following equation was applied:

$$
\begin{equation*}
b_{i, j}=\phi\left(\frac{d_{i}}{d_{j}}\right)^{\gamma}+(1-\phi)\left(\frac{d_{i}}{d_{j}}\right)^{\beta} \tag{2}
\end{equation*}
$$

Using Statistica ${ }^{\circledR}$ and the software developed at the Department of Process Equipment, Łódź Technical University, correlation equations of changes in the grinding rate as a function of particle size fraction $d_{i}$ were proposed. In the correlation the effect of particular size fractions as well as filling the mill with feed and grinding elements on the rate $S_{\mathrm{i}}$ was specified. A function in the form was selected:

$$
\begin{equation*}
S(i)=K_{S} \cdot d_{S}(i)^{n s} \cdot e^{-b s \cdot d s(i)} \tag{3}
\end{equation*}
$$

Table 2. Mass fractions of gabbro particles of various sizes. Ball mass 1.683 kg

| $\mathrm{d}_{\text {i }}$ | Grinding time [min], fractions $\mathrm{w}_{\mathrm{i}}$ [\%] of various particle sizes |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.5 | 1 | 3 | 5 | 7 | 10 | 15 | 20 | 30 | 40 | 60 | 80 | 110 |
| 2 | 25.8 | 23.1 | 20.1 | 19.1 | 17.5 | 14.8 | 13.1 | 12.6 | 10.4 | 7.7 | 5.4 | 3.3 | 0.5 |
| 1.6 | 13.2 | 11.5 | 11.4 | 8.7 | 8.2 | 7.1 | 6.6 | 4.9 | 3.3 | 2.7 | 3.3 | 2.7 | 0.0 |
| 1.4 | 12.1 | 12.1 | 10.9 | 10.4 | 8.7 | 7.7 | 4.9 | 4.4 | 4.4 | 3.3 | 2.7 | 2.2 | 1.6 |
| 1.25 | 11.5 | 12.1 | 10.9 | 9.8 | 8.7 | 8.7 | 8.2 | 7.7 | 6.6 | 5.5 | 3.8 | 3.3 | 1.6 |
| 1 | 23.1 | 22.5 | 20.7 | 18.0 | 16.9 | 15.9 | 14.8 | 12.0 | 10.4 | 8.7 | 8.2 | 4.9 | 2.2 |
| 0.8 | 8.2 | 9.3 | 11.4 | 13.7 | 13.1 | 11.5 | 10.9 | 10.4 | 9.8 | 9.8 | 8.7 | 9.2 | 10.3 |
| 0.63 | 2.2 | 2.8 | 3.3 | 3.8 | 3.8 | 4.4 | 4.9 | 5.5 | 6.0 | 6.6 | 6.5 | 7.1 | 7.0 |
| 0.5 | 0.6 | 1.1 | 2.2 | 3.3 | 4.4 | 5.5 | 6.0 | 6.6 | 7.1 | 7.7 | 9.2 | 10.3 | 11.4 |
| 0.4 | 1.7 | 2.2 | 2.7 | 3.3 | 4.4 | 5.5 | 6.0 | 6.0 | 6.6 | 7.1 | 7.1 | 8.2 | 9.7 |
| 0.315 | 0.6 | 0.6 | 0.5 | 1.1 | 1.6 | 1.6 | 1.6 | 1.6 | 2.2 | 2.7 | 3.3 | 4.4 | 6.5 |
| 0.2 | 0.0 | 0.6 | 1.1 | 1.6 | 2.2 | 2.7 | 3.8 | 4.4 | 4.9 | 5.5 | 6.5 | 7.6 | 9.2 |
| 0.125 | 0.0 | 0.6 | 1.1 | 1.6 | 2.7 | 4.9 | 6.6 | 8.2 | 8.7 | 9.8 | 10.3 | 10.9 | 12.4 |
| 0.09 | 0.6 | 1.1 | 2.2 | 2.7 | 4.4 | 6.0 | 7.1 | 8.7 | 9.8 | 11.5 | 12.0 | 12.5 | 13.5 |
| 0.071 | 0.0 | 0.0 | 0.5 | 1.1 | 1.6 | 2.2 | 3.3 | 4.4 | 6.6 | 7.7 | 8.7 | 9.2 | 9.7 |
| 0.063 | 0.6 | 0.6 | 0.5 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 1.1 | 1.1 | 1.1 |
| 0 | 0.0 | 0.0 | 0.5 | 1.1 | 1.1 | 1.1 | 1.6 | 2.2 | 2.7 | 3.3 | 3.3 | 3.3 | 3.2 |

Coefficients $K_{\mathrm{S}}, n_{\mathrm{s}}$, and $b_{\mathrm{s}}$ for each measuring series are given in Table 3. The obtained values of $S_{i}$ are also illustrated graphically. For easy interpretation of the results diagrams were prepared separately for gabbro (Fig.1) and sandstone (Fig. 2).

Table 3. Comparison of coefficients $\gamma, \beta$ and $\phi$ in equation (2) and $K_{\mathrm{s}}, b_{\mathrm{s}}$ and $n_{\mathrm{s}}$ in Eq. 3

| Ball load [pcs / kg] | Feed | $\gamma$ | $\beta$ | $\varphi$ | $\mathrm{K}_{\text {s }}$ | $\mathrm{b}_{\text {s }}$ | $\mathrm{n}_{\text {s }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 14/0.901 | Sandstone | 0.672 | 2.233 | 1.124 | 6.3 | 2.55 | 1.21 |
| 19/1.15 |  | 0.5625 | 4.894 | 1.738 | 11.9 | 2.13 | 2.23 |
| 23/1.392 |  | 0.752 | 2.335 | 1.607 | 1.06 | 1.4 | 0.29 |
| 27/1.683 |  | 0.8961 | 4.681 | 4.802 | 5.15 | 1.9 | 1.52 |
| 14/0.901 | Gabbro | 0.5439 | 4.592 | 0.9134 | 0.0358 | 1.12 | 0.645 |
| 19/1.15 |  | 0.7456 | 1.03 | 1.022 | 0.0234 | 0.1 | -0.319 |
| 23/1.392 |  | 0.6523 | 1.647 | 2.914 | 1.06 | 1.59 | 2.96 |
| 27/1.683 |  | 0.6834 | 2.917 | 3.217 | 0.23 | 1.1 | 0.733 |

It is interesting to follow the variability of $S_{\mathrm{i}}$ values for ground minerals. Owing to the fact that the grinding balls had a constant diameter, the impact of grinding media on the feed was similar. For gabbro particles (Fig. 1) from the upper size range, exceeding 1.4 mm , the impact of balls was similar. In this connection, the obtained values of comminution $S_{i}$ depend mainly on the number (mass) of grinding elements. The biggest gabbro particles are characterised by the lowest susceptibility to grinding. So, in the above mentioned range of particle sizes, the number of contact points does not play any important role.


Fig. 1. Change in grinding rate $S_{\mathrm{i}}$ of gabbro for the tested ball composition. Correlation function (Eq. 3)
The mean range of particle diameters (from around 0.5 to ca. 1.4 mm ) is characterised by the biggest differentiation of $S_{\mathrm{i}}$ values (Fig. 1). The effect of an increased number of contact points (increased mass of balls) is enhanced by a bigger variety of particles. The rate of grinding of hard inclusions still depends on the number of balls, but disruption of particle structure accelerates the rate of grinding. For very fine fractions low values of $S_{\mathrm{i}}$ were obtained and although differences of absolute values for a different number of balls are small, these absolute values are very significant. Additionally, the process of particle agglomeration causes that the rates of comminution of the smallest particles at a small number of balls (a few contact points) are close to zero, and even at the smallest size fractions the calculated values are negative.


Fig. 2. Change in grinding rate $S_{\mathrm{i}}$ of sandstone for the tested ball composition.Correlation function (Eq. 3)

For sandstone, the rates of grinding $S_{\mathrm{i}}$ are similar to those for gabbro. However, due to different particle structure - bigger fraction of soft inclusions in the matrix, the impact of grinding media on the feed causes a multiple increase of the grinding rate. This is caused by numerous soft calcium inclusions which bond harder particles of iron compounds. In the range of particle size fractions exceeding 1.6 mm , the obtained values of $S_{\mathrm{i}}$ for sandstone are more than $1.4 \mathrm{~min}^{-1}$ for the biggest number of balls (series $D$ ), and over $0.4 \mathrm{~min}^{-1}$ for the smallest number of balls (series A). For a comparable size fraction of gabbro particles the $S_{\mathrm{i}}$ rates are ca. $0.06 \mathrm{~min}^{-1}$ (series $D$ ) and $0.01 \mathrm{~min}^{-1}$ (series A), respectively.

Figures 3 and 4 show additionally changes of mean particle size of ground material in time. However, a comparison of the obtained curves does not allow us to draw conclusions like in the above analysis concerning the effect of the number of balls on the time required to obtain sufficient feed comminution. While for gabbro an increase of the number of balls causes shortening of the grinding time (series D ) to ca. 120 minutes, at grinding times for series A and B reaching ca. 200 min , in the case of sandstone the effect of the number of balls is not so obvious any longer. For series B the grinding times were the longest ( ca .16 min ), however by several minutes longer as compared to series A. Lack of grinding time increase for series A can be explained by the effect not only of the number of contact points but also a specific feature (nonhomogeneity) of sandstone particles. The effect of the number of grinding elements and their mass should be confirmed by further studies and determination of mechanical strength of particular size fractions.


Fig. 3. Change of mean size $d_{\mathrm{S}}$ of gabbro particles


Fig. 4. Change of mean size $d_{\mathrm{S}}$ of sandstone particles

## CONCLUSIONS

1. The following conclusions can be drawn from the obtained results:
2. The process of grinding of the tested minerals is determined by material characteristics, mainly particle structure and proportions between hard and soft components of the particles.
3. An increase of the number of balls causes an enhancement of grinding process, especially of mean particle size fractions.
4. The rate of comminution of small particle fractions depends on the number of contact points and particle agglomeration.

## NOMENCLATURE

$d_{s i}$ - mean (arithmetic) particle size in size fraction $i$
$b_{i, j}$ - distribution function defined as this part of ground material from size fraction $j$ which is transferred to size fraction $i$,
$b_{S}$ - coefficient of exponent in equation (3)
$b_{s}, n_{s} K_{s}$, parameters of correlation equation(3)
$d i, d j$ - particle diameters in size fractions $i$ and $j$, respectively
$S_{i}, S_{j}$ - specific rate of grinding of particles from size fraction $i$ or $j$ called also distribution parameter,
$w_{i}(t) w_{j}(t)$ - weight fraction of particles $i$ or $j$ after grinding time $t$,
$\beta, \varphi, \gamma$ - parameters in equation (2).

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W artykule przedstawiono wyniki badań, których celem było określenie szybkości przemiału surowców skalnych o zróżnicowanej podatności na rozdrabnianie. Próby mielenia przeprowadzono na surowcach skalnych, którymi było gabro oraz piaskowiec. Przemiał prowadzono w młynie kulowym w skali laboratoryjnej. W trakcie przemiałów zmieniano liczbę (masę) kul o stałej średnicy przy stałej masie nadawy. Podstawowym celem prowadzonych badań było określenia szybkości właściwej rozdrabniania poszczególnych frakcji rozmiarowych materiałów skalnych wykorzystywanych w budownictwie drogowym. Jako równanie wyjściowe zastosowano równanie Gardnera i Austina, w formie różniczkowej dla dyskretnych wartości udziałów, zakładając idealne wymieszanie mielonego materiału. Szybkości przemiału wybranych frakcji rozmiarowych mielonego materiału opisano w formie równania korelacyjnego. Określono parametry tego równania korelacyjnego oraz wplyw liczby (masy) kul na ich wartość. Dodatkowo, określono w postaci zależności matematycznej zmianę w czasie średniego wymiaru ziarna.


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